Binomial Probability Activity

By: Erin Bendul

Goal:

- The students will understand the concept of theoretical vs. experimental probability during a hands on activity, and apply their knowledge of binomial probability in future learning.

Objectives:

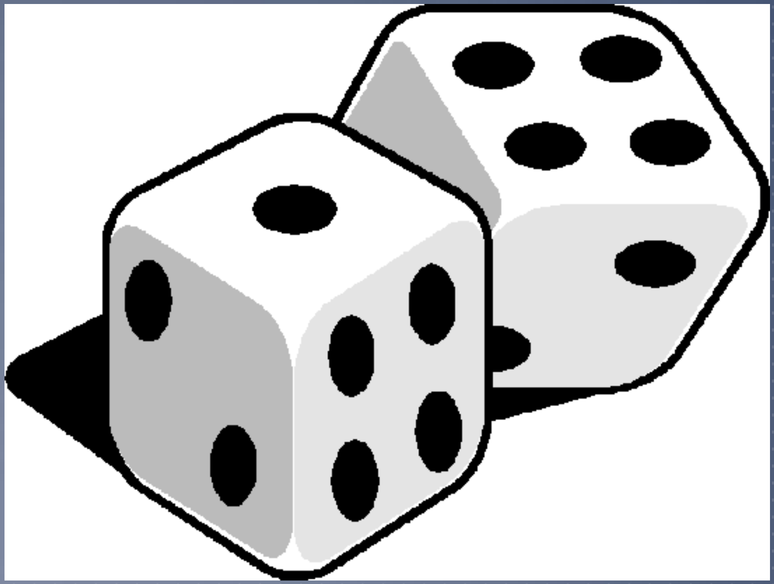
- Given a coin and a cube, students will find the theoretical probability of a specific outcome, and then perform the experiment to find the experimental probability.
- Students will collect data to find the percent of success vs. the percent of failure, and use this data to determine the theoretical probability of a specific event occurring, using the Bernoulli trails. Then, students will perform the experiment in order to determine the experimental probability of the same event occurring.

Standards:

- Expectations: In grades 9–12 all students should—
  - Know the characteristics of well-designed studies, including the role of randomization in surveys and experiments
  - Use simulations to explore the variability of sample statistics from a known population and to construct sampling distributions;
  - Compute and interpret the expected value of random variables in simple cases;
  - Understand the concepts of conditional probability and independent events;
  - Understand how to compute the probability of a compound event.

Materials:

- dice
- coin
- data chart



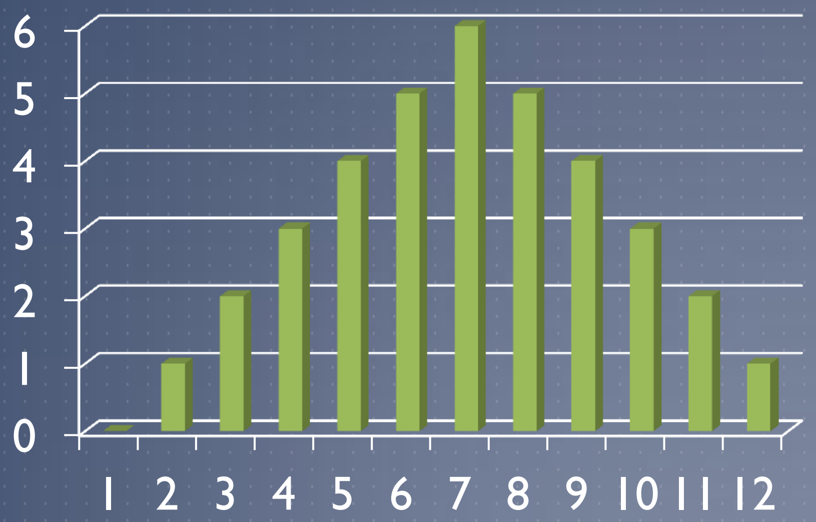
Pre-Activity

- Before beginning the lesson you should help students recall prior knowledge related to theoretical and experimental probability. Furthermore, you should reintroduce the formula used for Bernoulli probability.

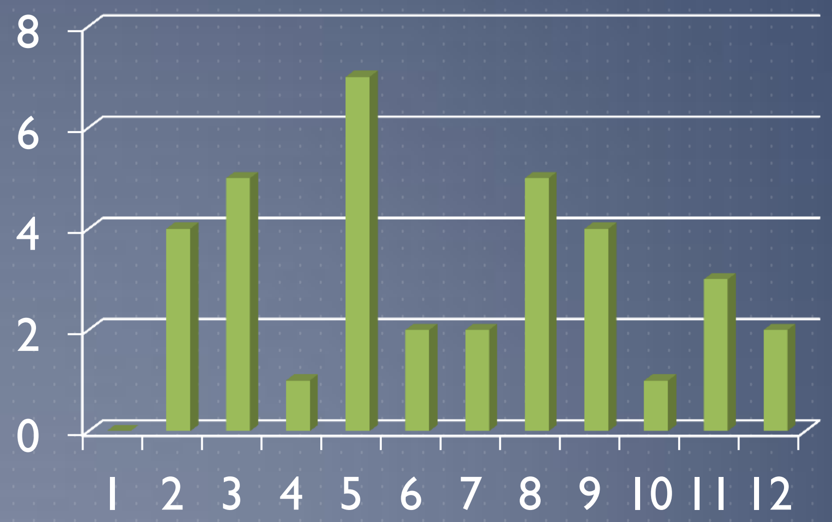
Theoretical & Experimental Probability

- Theoretical Probability is the probability that is calculated using math formulas. This is the probability based on math theory.
- Experimental Probability is calculated when the actual situation or problem is performed as an experiment. In this case, you would perform the experiment, and use the actual results to determine the probability. To accurately perform an experiment, you must : (i) identify what constitutes as a trial, (ii) perform a minimum number of 25 trials and (iii) set up an organizer to record your data.

Theoretical vs. Experimental



Theoretical Probability of rolling two dice and finding the sum of their faces



Experimental Probability of rolling two dice and finding the sum of their faces

Procedure for first hands on activity:

- Separate the class into groups of between 3-4 students.
- Assign each group a problem where they must calculate the theoretical probability of flipping a coin and rolling a dice to arrive at a specific outcome.
- Once students have determined the theoretical probability for their event occurring, distribute one die and one coin to each group.

Procedure for first hands on activity:

- Give each group a data collecting sheet and have the members of the group assign each other a specific task. (someone to record data, someone to roll the dice, someone to flip the coin)
- Ask students to perform 25 trails and record the information.
- Inform students that a trial consists of flipping a coin once and rolling a dice once.
- Ask students to compare their findings related to the theoretical and the experimental probability occurring.
  - Note: Every experiment will be different, therefore making the experimental probability different.

Example:

- A coin is tossed and a six-sided die is rolled. Find the probability of getting a head on the coin and a 6 on the die.
  - Theoretical Probability
    - Lets find the probability of each independent event occurring:
    - P(heads) = $\frac{1}{2}$ There is only 1 head on a coin and there are two total outcomes (heads or tails)
    - P(6) = $\frac{1}{6}$ There is only one 6 on a die and there are 6 total outcomes on a die (1,2,3,4,5,6)
    - P(heads and a 6)= $\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$
  - The theoretical probability of flipping a head on a coin and rolling a 6 on a die is $\frac{1}{12}$ or 8.3%.

Example Continued:

- Experimental Probability
  - Identify a trial: A trial consists of flipping a coin once and rolling a die once
  - Conduct 25 trials and record your data

Coin	T	T	T	H	T	T	T	T	T	H	T	T	T	T	T	H	H	H	H	T	H	T	H	T	H
Die	2	4	4	2	4	5	4	5	5	1	4	4	3	3	3	6	2	3	5	4	3	2	1	2	3
H and 6	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	Y	N	N	N	N	N	N	N	N	N

For the above table, H represents heads, T represents tails, and the numbers represents the number which was face up on the die. In this experiment, there was only 1 trial out of 25 where a head was flipped on the coin and a 6 was rolled on the die. This means that the experimental probability is $\frac{1}{25}$ or 4%.

Conclusions:

- The theoretical probability is 8.3% and the experimental probability is 4%.
- We have determined the theoretical probability and the experimental probability are not equal, however there is not a significant difference.
- Note: In most experiments, the theoretical and experimental probabilities will not be equal, however they should be relatively close.

Procedure for Second Hands on Activity:

- Explain to students that in order to conduct an activity using Bernoulli Trials properly, they must collect data to use.
  - Their data will represent the probability of success and the probability of failure.
- Next, have students use this data to determine the theoretical probability a particular event will occur using the Bernoulli Formula.

Bernoulli Trial

$$P(k) = \binom{n}{k} p^k q^{n-k}$$

Where

n= the number of trials

k= 0,1,2,…n which represents what you are looking for

p= the probability of success in a single trial

q= the probability of failure in a single trial or (1-p)

P(k)= gives the probability of successes in *n* binomial trials

Note: $\binom{n}{k}$ can be calculated by doing $\frac{n!}{k!(n-k)!}$

Example using Bernoulli Trial

- You are sitting in the lobby of your school waiting for your next class to begin. What is the probability that out of the next 100 people who walk through the door, exactly 20 of them will be wearing a shirt or sweatshirt with the MSMC logo printed on it?

Steps:

- Before beginning any calculations, students must collect data related to the example.
- Have students find the probability of success and failure to use during their calculations.
- Ask students to record how many of their peers enter the cafeteria wearing the school logo on any given day.
- To make this number more accurate, perhaps try having three trials, and finding the average of the three trials.
    - Let's say for the sake of our example, that out of 100 students who entered the cafeteria for lunch, 23 of them were wearing the school logo. This means that 77 of them were not.

Steps:

- Tell students to use the data collected from their observation to determine the probability that exactly 20 out of 100 students will be wearing the MSMC logo.

$$P(k)= \binom{n}{k} p^k q^{n-k}$$

$$P(20)= \binom{100}{20}(.23)^{20}\,(.77)^{100-20}$$

$$P(20)= \binom{100}{20}(.23)^{20}\,(.77)^{80}$$

$$P(20)= 0.0763776074$$

Expanding on the lesson:

- Find the probability that at most 1 out of 3 students will be wearing an MSMC logo
- Find the probability that at least 2 out of 3 students will be wearing an MSMC logo

At most 1 out of 3

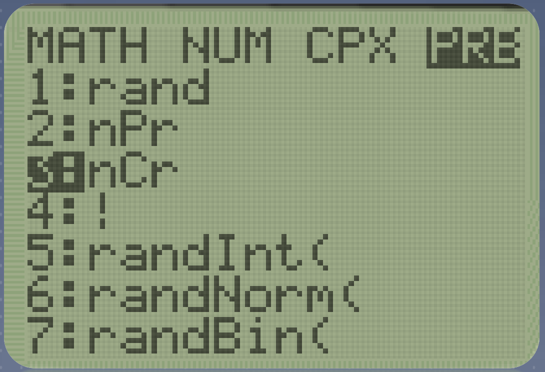
$$[\binom{3}{0}(.23)^0(.77)^3] + [\binom{3}{1}(.23)^1(.77)^2] = .86534$$

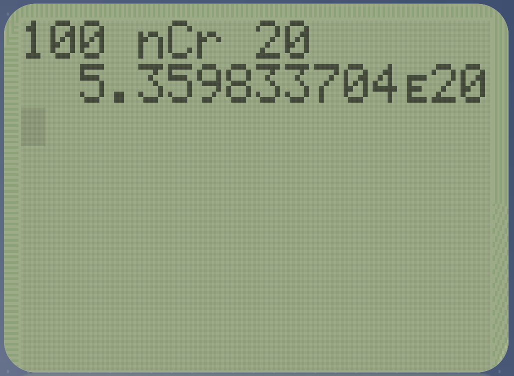
At least 2 out of 3

$$[\binom{3}{2}(.23)^2(.77)^1] + [\binom{3}{3}(.23)^3(.77)^0] = .134366$$

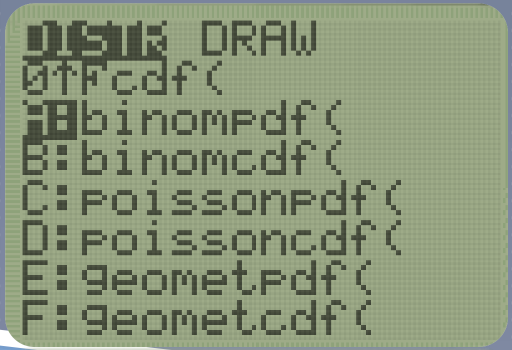
On calculator:

To calculate nCr, enter the total number of trials, press the MATH button, scroll to PRB down to 3 (nCr) , enter the number you are looking for, and press enter.





To find the probability of an event occurring, enter 2nd VARS (DISTR), scroll down to A:binoMPDF(and enter n, p, and k respectively.



Other options:

- school colors
- high-school football game
- Field Day
- students wearing head phones
- students wearing backpacks
- students wearing UGGS

Conclusions:

- Students will see that in order to find the theoretical probability they must collect data
  - Their data will represent the probability of success and the probability of failure
  - The probability will not always be 50/50
- The Bernoulli Formula can be used to determine the probability of various events occurring
- When dealing with “at most” and “at least” students must calculate multiple equations and add them together
  - They are compliments of each other, which add to 1.